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SUPPRESSION OF SPONTANEOUS FLUCTUATIONS IN 2n-TERMINAL AMPLIFIERS AND NETWORKS

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Zusammenfassung

Diese Arbeit knüpft an früheren Arbeiten über die Verringerung der Wirkung spontaner Schwankungen in Verstärkern an, indem diese als $2n$-Pole dargestellt werden. Im 2. Abschn. wird die Anwendung von Rückkopplung beliebigen Vorzeichens auf lineare Sechspole betrachtet und werden Ausdrücke für die spontanen Spannungsschwankungen und Signalspannungen an den verschiedenen Ausgängen gewonnen. Im Abschn. 3 ergibt sich hieraus u.A. der Satz: das Verhältnis der Schwankungsspannung zur Signalspannung kann an jedem Ausgang durch Rückkopplung auf den kleinsten Wert herabgedrückt werden, der vor der Rückkopplung an irgend einem Ausgang gilt. Abschn. IV beschäftigt sich mit dem Einfluss verschiedener Korrelationen zwischen den Schwankungen an den Ausgängen auf die günstigsten Ergebnisse welche mit einer Rückkopplung erzielt werden können. Im Abschn. V werden Anwendungsbeispiele erörtert. Abschn. VI beschäftigt sich mit linearen Achtpolen und mit der Anwendung von Rückkopplung zur Verringerung der spontanen Schwankungen im Verhältnis zum Signal. Abschn. VII enthält die Erweiterung dieser Ergebnisse auf lineare $2n$-Pole.

I. *Introduction.* In a previous article [2]) we have shown, that the signal to noise ratio at the output of a four-terminal amplifier cannot be altered by applying positive or negative feed-back. In many practical cases, however, the amplifiers have more than four terminals. Well known examples are afforded by modern multigrid amplifier valves. We have already published various examples, in which the application of feedback resulted in a suppression of the spontaneous fluctuations in those valves [1]). In the present article we give a general theoretical framework, covering those examples and affording new possibilities. As many of the formulas used here may be derived in close analogy with the formulas in the case of four-

terminal networks, no detailed calculations will be given here.

II. *Feedback in six-terminal networks.* We first will consider a six-terminal network for amplifying purposes (fig. 1). The input signal is applied to the input terminals 1 and 2. The two pairs of output

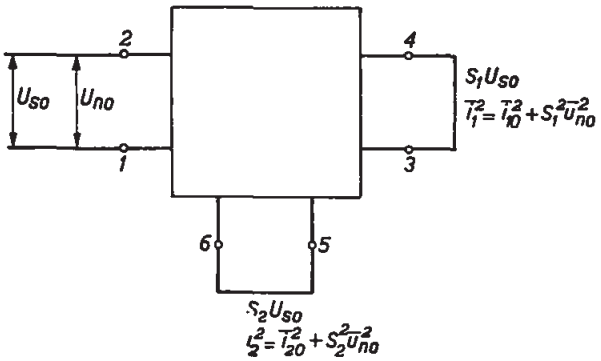

Fig. 1. Six terminal amplifier with the input terminals 1, 2 and the two pairs of output terminals 3, 4 and 5, 6. Without feedback the input signal voltage is U_{s0} and the output signal currents are $S_1 U_{s0}$ and $S_2 U_{s0}$ respectively. The input fluctuation (or noise) voltage is in that case u_{n0}. The output noise currents are $\overline{i_1^2}$ and $\overline{i_2^2}$ as indicated at the terminals (3, 4) and (5, 6).

terminals (5, 6) and (3, 4) may be considered as short-circuited, which simplifies the matter without loss of generality. If we make feedback connections from the output (5, 6) to the input and then calculate the signal to noise ratios of the output currents between 5 and 6, we will find them unaltered by the feedback. The same holds for the output terminals (3, 4). This may be inferred from the general theorem on feedback in four-terminal networks, derived in a previous publication [2]).

We now consider the alteration of the signal to noise ratio corresponding to the output (3, 4), due to a feedback from the output (5, 6) to the input 1, 2. The input signal prior to the feedback is U_{s0} and the input noise voltage in a small frequency interval Δf is denoted by u_{n0}. After the feedback the corresponding values are U_s and u_n. The feedback will be represented by a quantity β having the dimension of an impedance, according to the relation:

$$U_s = U_{s0} + \beta S_2 U_s,$$

or

$$U_s = \frac{U_{s0}}{1 - \beta S_2},$$

where S_2 is the ratio of the output current at the terminals (5, 6) to the input voltage at the terminals (1, 2).

Before the feedback the signal current between the connections 3 and 4 is $S_1 U_{s0}$ and the noise current $\overline{i_1^2} = \overline{i_{10}^2} + S_1^2 \overline{u_{n0}^2}$, whereas with feedback these currents are $S_1 U_s$ and:

$$\overline{i_{n1}^2} = S_1^2 \frac{\overline{u_{n0}^2}}{(1 - \beta S_2)^2} + \overline{\left[S_1 \frac{i_{20}\beta}{(1 - \beta S_2)} + i_{10}\right]^2},$$

in close analogy with the case of a four-terminal network. Here S_1 is the ratio of the output current at the terminals (3, 4) to the input voltage at the terminals (1, 2), whreas i_{10} and i_{20} denote the output fluctuation currents in a small frequency interval Δf at the connections (3, 4) and at the connections (5, 6) respectively, if the input of the six-terminal network is short-circuited. In this expression some correlation may exist between the fluctuation currents i_{20} and i_{10}. The noise to signal ratio at the output terminals 3 and 4 has the value:

$$[\text{Ratio } (3, 4)]^2_{\substack{\\ \textit{feedback}}} = \frac{\overline{u_{n0}^2} + \overline{\left[i_{20}\beta + \frac{i_{10}}{S_1}(1 - \beta S_2)\right]^2}}{U_{s0}^2}. \tag{1}$$

If Z_{i0} is the input impedance of the amplifier without feedback and Z_i the input impedance of the amplifier after application of feedback, Z_i is given by the relation [2]):

$$Z_i = \frac{Z_{i0}}{1 - \beta S_2}. \tag{2}$$

The obtained noise to signal ratio is to be compared with the corresponding ratios at the output terminals (3, 4) and (5, 6) respectively, before applying feedback (see fig. 1):

$$[\text{Ratio } (3,4)]^2_{\substack{\\ \textit{no feedback}}} = (\overline{u_{n0}^2} + \overline{i_{10}^2}/S_1^2)/U_{so}^2; \tag{3}$$

$$[\text{Ratio } (5, 6)]^2_{\substack{\\ \textit{no feedback}}} = (\overline{u_{n0}^2} + \overline{i_{20}^2}/S_2^2)/U_{s0}^2. \tag{4}$$

Hence, using the terminals (3, 4) as output, the noise to signal ratio will have been lowered, if (see eqs. 1 and 3):

$$u^2 < \overline{i_{10}^2}/S_1^2, \tag{5}$$

where u^2 is defined as:

$$u^2 = \overline{\left[\frac{i_{20}}{S_2}\beta S_2 + \frac{i_{10}}{S_1}(1 - \beta S_2)\right]^2}. \qquad (6)$$

Obviously U_{s0}^2/u^2 expresses the signal to noise ratio at the output, if a fluctuation-free signal is applied to the input of a fluctuation-free amplifier.

In sections III and IV we consider the cases, in which eq. (5) is satisfied.

III. *Reduction in noise to signal ratio in six-terminal amplifiers by applying feedback.* If we choose the feedback such that:

$$1 - \beta S_2 = 0 \text{ (therefore } Z_i = \infty),$$

the output noise to signal ratio according to eq. (1) will be exactly equal to the ratio, expressed by eq. (4). Hence, if this ratio of eq. (4) is smaller than the ratio of eq. (3), feedback leads to a relative suppression of the noise with respect to the signal for the output terminals (5, 6).

An amplifier with more than 2 output terminals may, according to this result, by applying proper feedback, attain a noise to signal ratio equal to the smallest noise to signal ratio valid at one of the output terminal-pairs before the feedback.

In general, the quantity βS_2, which characterizes the feedback, will be complex: $\beta S_2 = x + jy$, where $j = +\sqrt{-1}$. We may show, *that the signal to noise ratio connected with a complex value of the feedback factor βS_2 is always smaller than the corresponding ratio connected with a real value of βS_2, equal to the real part of the former complex value.* Considering the quantity u^2 we have, for $\beta S_2 = x + jy$:

$$u^2 = \overline{\left[\frac{i_{20}x}{S_2} + \frac{i_{10}(1-x)}{S_1}\right]^2} + \overline{\left[\frac{i_{20}}{S_2} - \frac{i_{10}}{S_1}\right]^2} y^2.$$

The second expression to the right is always positive, if i_{20} and i_{10} are not completely correlated, and zero, if i_{20} and i_{10} are completely correlated and $i_{20}/S_2 = i_{10}/S_1$. This obviously proves the above proposition. It may also be formulated as follows: If feedback is applied to a six-terminal amplifier, the noise to signal ratio may always be reduced by annihilating the complex part of the feedback factor βS_2.

The problem of obtaining the lowest possible noise to signal ratio lies in determining the smallest value which eq. (6) can ever attain

as a function of βS_2. This problem together with the influence of correlation between i_{10} and i_{20} on it will be discussed in Section IV.

IV. *Influence of correlation between the fluctuations at the two pairs of output terminals.* In the first place we consider the case, that i_{10} and i_{20} are in no way correlated. When the quantity α^2 is defined by the relation:

$$\alpha^2 = \frac{\overline{i_{20}^2}}{S_2^2} \Big/ \frac{\overline{i_{10}^2}}{S_1^2},$$

the quantity u^2 attains a minimum value for $\beta S_2 = 1/(1 + \alpha^2)$. Inserting this in eq. (6) yields:

$$u^2 = \frac{\overline{i_{10}^2}}{S_1^2} \frac{\alpha^2}{1 + \alpha^2} = \frac{\overline{i_{20}^2}}{S_2^2} \frac{1/\alpha^2}{1 + 1/\alpha^2}, \tag{6a}$$

(according to the definition of α^2), which is always less than $\overline{i_{10}^2}/S_1^2$. Hence, according to eq. (3) and (1), the noise ratio is reduced by applying feedback in this way. This result is also better than that, which can be obtained according to the first theorem of Section III.

If $\alpha^2 > 1$, the fluctuations proper to the output terminals (3, 4) are less than those, connected with the terminals (5, 6). If, on the other hand, $\alpha^2 < 1$, the fluctuations at the terminals (5, 6) are smaller than those at (3, 4). In the former case it will offer advantages, if we use (3, 4) as output- and (5, 6) as feedback-terminals. If the latter condition prevails, these terminals sometimes may better be used in a reversed way, i.e. (5, 6) as output- and (3, 4) as feedback-terminals. If, however, the lowest noise to signal ratio is obtained in both cases by applying suitable feedback, the noise to signal ratio will be the same.

Taking two identical four-terminal amplifiers and connecting their input terminals in parallel, suitable feedback may result in a reduction of u^2 by a factor 2 (eq. 6*a*), because $\alpha^2 = 1$. As eq. (1) also contains $\overline{u_{n0}^2}$, the reduction in noise to signal ratio, according to eq. (1), is somewhat less than a factor 2.

Secondly we assume complete correlation between the fluctuating currents i_{10} and i_{20} and take:

$$\alpha = \frac{i_{20}}{S_2} \Big/ \frac{i_{10}}{S_1}.$$

If the feedback factor βS_2 is real, the imaginary part of βS_2 being unimportant here (section III), the quantity u of eq. (6) is given by:

$$u^2 = \frac{\overline{i_{10}^2}}{S_1^2} \mid 1 \quad \beta S_2 + \alpha\beta S_2 \mid^2.$$

$u^2 = 0$, if $\beta S_2 = 1/(1 - \alpha)$. If $\alpha < 0$, we obtain positive values of βS_2, which are < 1 and hence we have negative feedback. If $\alpha > 1$, we have negative values of βS_2, and hence we have *positive feedback* (which means an increase of input voltage due to the feedback). If $\alpha = 1$, we obtain $u^2 = \overline{i_{20}^2}/S_2^2$ and hence no reduction of noise to signal ratio is possible. If, finally, $0 < \alpha < 1$, we obtain positive values of βS_2, which are > 1. As $Z_i = Z_{i0}/(1 - \beta S_2)$, the real part of Z_i will be negative, and therefore application of this kind of feedback may in some cases lead to instability of the amplifier and to the generation of oscillations. In that case, however, (3, 4) may be used as feedback terminals and (5, 6) as output terminals. Then u^2 will be zero, if $\beta S_1 = 1/(1 - \alpha^{-1})$ and because $0 < \alpha < 1$, $\alpha^{-1} > 1$ and therefore $\beta S_1 < 0$, resulting in a feedback, which will always be possible.

In conclusion, our results are: *If the fluctuation currents at the two pairs of output terminals are completely correlated, a reduction of the noise to signal ratio may be obtained by applying proper feedback from one pair of output terminals, using the other pair as output connections, if* $\alpha \neq 1$. *The two pairs of output terminals may of course also be used inversely. By properly choosing the amount of feedback, complete suppression of valve noise is possible.*

Thirdly, in the case of partial correlation between the fluctuation currents at the two pairs of output terminals, we may be brief. This case may be considered as a combination of the two cases, already mentioned, one corresponding to complete correlation and the other to no correlation. The results afford no new conclusions besides those, formulated above. Some results are already published in our first article on noise reduction in amplifying valves [1]).

V. *Examples of feedback in six-terminal amplifiers.* As a first example we consider a tetrode amplifier valve (fig. 2). Direct current connections have been omitted in fig. 2 for sake of simplicity. The three pairs of terminals, according to fig. 1, have been indicated in fig. 2. The fluctuation currents at (3, 4) and (5, 6) are partly correlated.

A useful kind of feedback from the terminals 5 and 6 to the input terminals 1 and 2 makes use of an inductivity between 5 and 6. By the capacity between g_2 and g_1, the voltage generated across this inductivity is fed back to the input. A substantial reduction of the noise to signal ratio at the output terminals 3 and 4 may be obtained in this way, if the inductivity is properly chosen [1]).

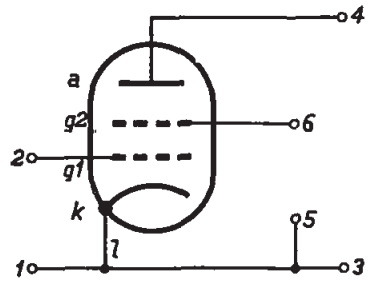

Fig. 2. Tetrode amplifier valve as a six terminal amplifier.

Of the various electrode leads carrying d.c. in fig. 2, i.e. the anode lead, the lead of g_2 and the cathode lead l, the ratio of fluctuation current to signal current is smallest for the latter lead l. Hence, by proper feedback from the cathode lead l, according to the first theorem derived in section III, we may reduce the noise to signal ratio at the output terminals 3 and 4 so as to equal the corresponding ratio in the cathode lead. This feedback may be achieved by cutting this lead and inserting a capacity between the two terminals thus obtained. The voltage across this capacity is fed back to the input by the capacity between the cathode k and the grid g_1. This simple means of noise suppression at the output may also be applied to mixer stages of superheterodyne receivers. As is well known, the noise to signal ratio at the output of such stages is often 50 or 100 times higher than at the output of well designed amplifier stages. By the above feedback, a reduction of this ratio (square of fluctuation current to square of signal current)e.g. by a factor 50 may be obtained, hence practically making multigrid-valve mixer stages equal to a low-noise amplifier stage, as regards noise to signal ratio at the output.

VI. *Feedback in 8-terminal amplifiers.* We consider an 8-terminal amplifier as shown in fig. 3. We again assume the output terminals (3, 4), (5, 6) and (7, 8) to be short-circuited, which implies no loss of generality. If an input signal U_s is applied to the terminals (1, 2)

(S_1, S_2 and S_3 have the same physical meaning as S_1 and S_2 in section II), output currents S_1U_s, S_2U_s and S_3U_s flow at the three pairs of output terminals, as shown in fig. 3. We apply feedback from the

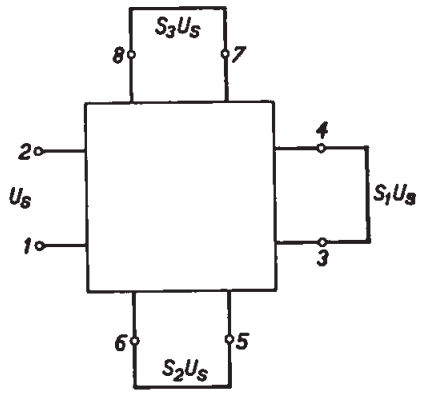

Fig. 3. Eight terminal amplifier with input signal voltage U_s and output signal currents $S_1\,U_s$, $S_2\,U_s$ and $S_3\,U_s$ respectively.

terminals (5, 6) to the input terminals (1, 2) as well as from the terminals (7, 8) to the input terminals (1, 2). These feedback connections, in close analogy to our above reasoning for a six-terminal amplifier, are expressed by the relation:

$$U_s = U_{s0} + \beta_2 S_2 U_s + \beta_3 S_3 U_s,$$

where β_2 and β_3 are quantities having the dimension of an impedance, U_{s0} is the input signal voltage before applying feedback and U_s the input signal voltage after applying feedback. We hence obtain:

$$U_s = \frac{U_{s0}}{1 - \beta_2 S_2 - \beta_3 S_3}.$$

If again Z_{i0} and Z_i are the input impedance without and with feedback respectively, Z_i is given by $Z_{i0}/(1 - \beta_2 S_2 - \beta_3 S_3)$. The input fluctuation voltage at the terminals (1, 2) before applying feedback in a small frequency interval Δf is again u_{n0} and this is altered by the feedback specified above into $u_{n0}/(1 - \beta_2 S_2 - \beta_3 S_3)$. Moreover at the three pairs of output terminals (3, 4), (5, 6) and (7, 8) we have spontaneous fluctuation currents in the case of short-circuited input terminals (1, 2), amounting to i_{10}, i_{20} and i_{30} respectively in a small frequency interval Δf. These fluctuations i_{20} and i_{30} are also fed back

to the input terminals. Finally, we obtain for the noise to signal ratio at the output terminals (3, 4) with feedback:

$$\frac{\overline{u_{n0}^2} + \overline{\left[\frac{i_{10}}{S_1}(1 - \beta_2 S_2 - \beta_3 S_3) + \frac{i_{20}}{S_2}\beta_2 S_2 + \frac{i_{30}}{S_3}\beta_3 S_3\right]^2}}{U_{s0}^2},$$

whereas it is without feedback:

$$\frac{\overline{u_{n0}^2} + \overline{i_{10}^2}/S_1^2}{U_{s0}^2}.$$

Thus we obtain a reduction of noise to signal ratio by means of the specified feedback, if the quantity:

$$v^2 = \overline{\left[\frac{i_{10}}{S_1}(1 - \beta_2 S_2 - \beta_3 S_3) + \frac{i_{20}}{S_2}\beta_2 S_2 + \frac{i_{30}}{S_3}\beta_3 S_3\right]^2} \qquad (7)$$

is smaller than the corresponding value without feedback, i.e. if:

$$v^2 < \frac{\overline{i_{10}^2}}{S_1^2}. \qquad (8)$$

We draw attention here to the close analogy between the quantity v^2 according to eq. (7) for an 8-terminal amplifier and the quantity u^2 according to eq. (6) for a 6-terminal amplifier.

By discussing eq. (7) and the condition (8) for the reduction of the noise to signal ratio by feedback, some of the results obtained in sections III and IV for 6-terminal amplifiers may be shown to retain their validity for 8-terminal amplifiers. Thus, e.g. the reduction of noise to signal ratio obtained by applying complex values of $\beta_2 S_2$ and of $\beta_3 S_3$ is always less than the reduction, obtained by applying real values of $\beta_2 S_2$ and $\beta_3 S_3$ equal to the real parts of the previous complex values.

The actual reduction of noise to signal ratio obtained depends on the correlation between the fluctuation currents at the three different output pairs of terminals.

We first assume completely uncorrelated fluctuation currents i_{10}, i_{20} and i_{30}. Writing:

$$\frac{\overline{i_{20}^2}}{S_2^2}\Big/\frac{\overline{i_{10}^2}}{S_1^2} = \alpha_2^2 \text{ and } \frac{\overline{i_{30}^2}}{S_3^2}\Big/\frac{\overline{i_{10}^2}}{S_1^2} = \alpha_3^2,$$

we obtain for the minimum of the quantity v^2 of eq. (7):

$$(v^2)_{min} = \frac{\overline{i_{10}^2}}{S_1^2}\,\frac{\alpha_2^2\alpha_3^2}{\alpha_2^2 + \alpha_3^2 + \alpha_2^2\alpha_3^2}. \qquad (7a)$$

As an example, take $\alpha_2^2 = \alpha_3^2 = 1$. Thus the specified feedback is leading to a reduction of v^2 by a factor 3 compared with the value without feedback. This may e.g. be realized by taking three identical separate four-terminal amplifiers and connecting their input terminals in parallel.

Secondly we assume complete correlation between the fluctuation currents i_{10}, i_{20} and i_{30}. Writing

$$\beta_2 S_2 = x_2, \quad \beta_3 S_3 = x_3, \quad \frac{i_{20}}{S_2} \Big/ \frac{i_{10}}{S_1} = \alpha_2, \quad \frac{i_{30}}{S_3} \Big/ \frac{i_{10}}{S_1} = \alpha_3,$$

the value v^2 of eq. (7) is then given by:

$$v^2 = \frac{\overline{i_{10}^2}}{S_1^2} [(1 - x_2 - x_3) + \alpha_2 x_2 + \alpha_3 x_3]^2. \tag{7b}$$

From (7*b*) we may conclude, that v^2 will be zero, if x_2 and x_3 can be chosen such, that:

$$(1 - x_2 - x_3) + \alpha_2 x_2 + \alpha_3 x_3 = 0, \tag{9}$$

which is always possible except for $\alpha_2 = \alpha_3 = 1$ (in that case no change in v^2 can be effected by applying feedback).

In a six-terminal amplifier the required amount of feedback was in some cases sufficient to produce instability. Now it may be seen, that in the case of an eight-terminal amplifier, instability will occur much less frequently than in the former case. As the input impedance Z_i is given by

$$Z_i = \frac{Z_{i0}}{1 - \beta_2 S_2 - \beta_3 S_3} = \frac{Z_{i0}}{1 - x_2 - x_3},$$

the amplifier will be stable, if:

$$(1 - x_2 - x_3) \geq 0. \tag{10}$$

Both relations (9) and (10) may be satisfied by properly choosing x_2 and x_3, except if $\alpha_2 = \alpha_3$. This would mean, that the signal to noise ratios in both pairs of feedback terminals are the same and because the fluctuations are completely correlated, we have in fact an eight-terminal amplifier behaving as a six-terminal one.

In an actual amplifier the fluctuation currents i_{10}, i_{20} and i_{30} will be neither completely correlated nor completely uncorrelated but will show a partial correlation. Those currents i_{10}, i_{20} and i_{30}, however, may be regarded as a combination of several completely correlated and completely uncorrelated components. In this case our

conclusions remain generally unaltered, and are quite similar to those mentioned in section IV for six-terminal amplifiers.

VII. *Extension to 2n-terminal amplifiers.* The set-up of the above treatment has been such as to permit its almost immediate extension to 2n-terminal amplifiers. In this way the reasoning pertaining to 6-terminal amplifiers has been extended above to 8-terminal amplifiers with only a few new points brought into the latter reasoning.

We only discuss an example of completely uncorrelated fluctuations, considering n identical separate amplifiers with the inputs connected in parallel and satisfying the condition of complete incorrelation of their output fluctuation currents. A fluctuation-free signal is supplied to the input terminals. *Then, by applying proper feedback from n—*1 *output pairs of terminals, as specified in section VI eqs.* (8) *and* (7*a*) *for the case of three amplifiers, the mean square noise to signal ratio at the output of the n-th amplifier may be reduced by a factor n.*

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REFERENCES

1) M. J. O. Strutt and A. van der Ziel, Physica **8**, 1, 1941.
2) M. J. O. Strutt and A. van der Ziel, Physica **9**, 513, 1942.
The second article contains an extensive bibliography of recent contributions in this field.